PART XV: SPECIAL LOADING CONDITIONS

15.1-LATERAL DISTRIBUTION OF A CONCENTRATED LOAD

15.1.1-Lateral Distribution of a Concentrated Load for Moment

The lateral distribution factors for moment in Table 15.1.1 have been part of the Specification since the 1944 edition. These factors, keyed to the nominal thickness of the flooring or decking involved (two to six inches thick) and the spacing of the stringers or beams in feet, S, are based on recommendations of the American Association of State Highway and Transportation Officials (2). The factors for moment are:

2 inch plank	S/4.0
4 inch nail-laminated	S/4.5
6 inch nail-laminated	S/5.0
structural concrete	S/6.0

(For all cases when S > N, where N is the denominator of the moment factors, load on adjacent stringers is based on the reactions of the load assuming flooring acts as simple beam)

The two-inch plank floor refers to one made of pieces of lumber laid edge to edge with the wide faces bearing on the supporting beams or stringers. The four-inch and six-inch laminated floors refer to those made of pieces of lumber laid face to face with the narrow edges bearing on the supporting beams or stringers, with each piece being nailed to the preceding piece (2). Nails typically penetrate into two adjacent pieces, are staggered and are alternated on the top and bottom edges (53). Flooring is typically attached to stringers by toe nailing.

The factors obtained from the foregoing S/N ratios apply to bridges designed for one traffic lane and to interior beams and stringers only. The computed factor gives the fraction of the wheel load (both front and rear of tractor or trailer axles on one side) positioned to give maximum bending moment at midspan of the beam or stringer closest to the wheel load (2,53).

When the average spacing of the beams or stringers in a one traffic lane bridge exceeds the denominator (N) of the ratio (S/N), the concentrated live load on the two beams or stringers adjacent to the applied load is taken as the reactions of the load assuming the flooring or decking between the beams or stringers is acting as a simple beam. Examples C15.1-1 and C15.1-2 illustrate these provisions.

Example C15.1-1

Single lane bridge with 2 by 6 in. laminated decking edge bearing on 10 by 24 in. stringers spaced 23 in. apart. Total load on front and rear trailer axles is 65,000 lb. What is the concentrated load to be used for calculation of the stringer bending moment associated with total load on the trailer axles?

S = 1.92 ft

From Table 15.1.1:

Lateral distribution factor = S/N
= 1.92/5.0
= 0.384

Applied wheel load = front and rear axles, (one side)
= 1/2 (65,000)
= 32,500 lb

Concentrated load, P = 0.384 (32,500)
(for bending moment calculation) = **12,480 lb acting at midspan of stringer**

The live load bending moment for outside beams or stringers is calculated using a load equal to the reaction of the wheel load assuming the flooring or decking between the outside and adjacent stringer is acting as a simple beam (2). This procedure is comparable to that used where S > N.

Lateral distribution factors determined in accordance with Table 15.1.1 can be used for any type of fixed or moving concentrated load. The lateral distribution factors determined from the table have been verified by field tests on five timber bridges ranging from 15 to 46 feet in span and by laboratory tests on three full-size bridge deck and stringer assemblies 16 to 28 feet in span (53). These tests indicate the factors are somewhat conservative, particularly at S/N ratios greater than 0.60.

For bridges of two or more traffic lanes, the American Association of State Highway and Transportation Officials (2) provides other lateral distribution factors.

Example C15.1-2

Consider the previous example but with stringers spaced 72 in. apart. What is the concentrated load to be used for calculation of the stringer bending moment associated with the 65,000 lb load on the trailer axles when the wheel load is 6 in. from the nearest stringer? 2 ft from the nearest stringer?

6 in. from stringer:

S/N = $6.0/5.0 > 1.0$

Reaction = $P\,(5.5/6.0)$
= $0.917\ P$

Applied wheel load = front and rear axles, (one side)
= 1/2 (65,000)
= 32,500 lb

Concentrated load, P = 0.917 (32,500)
(for bending moment calculation) = **29,792 lb acting at midspan of stringer**

2 ft from stringer:

S/N = $6.0/5.0 > 1.0$

Reaction = $P\,(4.0/6.0)$
= $0.667\ P$

Applied wheel load = 32,500 lb

Concentrated load, P = 0.667 (32,500)
(for bending moment calculation) = **21,678 lb acting at midspan of stringer**

15.1.2-Lateral Distribution of a Concentrated Load for Shear

The lateral distribution factors for shear in Table 15.1.2 have been provisions of the Specification since the 1944 edition. These factors relate the lateral distribution of concentrated load at the center of the beam or stringer span as determined under 15.1.1, or by other means, to the distribution of load at the quarter points of the span. The quarter points are considered to be near the points of maximum shear in the stringers for timber bridge design.

The tabulated values of the percentage of a concentrated load on the center beam at the quarter point of the span and the percentage of the same load on the center beam at midspan is closely described by the following relation:

$$P_{1/4} = -1.807 + 1.405 \log(P_m) \qquad \text{(C15.1-1)}$$

where:

$P_{1/4}$ = percentage of load at 1/4 point of center beam

P_m = percentage of load at midspan of center beam
= (S/N) from Table 15.1.1 or other basis

Values of $P_{1/4}$ from Table 15.1.2 are used to determine the actual shear stress from the wheel or other concentrated load being considered. Field and laboratory tests of full-size timber bridges verify the appropriateness of the Table 15.1.2 values and indicate they are conservative at S/N ratios above 0.50 (53).

15.2-SPACED COLUMNS

15.2.1-General

Background.

As used in the Specification, spaced columns refer to two or more individual members oriented with their longitudinal axis parallel, separated at the ends and in the middle portion of their length by blocking and joined at the ends by split ring or shear plate connectors capable of developing required shear resistance. The end fixity developed by the connectors and end blocks increases the load carrying capacity in compression parallel to grain of the individual members only in the direction perpendicular to their wide faces (parallel to the d_1 dimension in Figure 15A of the Specification).

Design provisions for spaced columns (62) have remained essentially unchanged since the 1944 edition of the Specification except for changes in the general column equations applicable to all wood columns (see Commentary for 3.7.1.5). In this regard, the original 1944 end fixity factors, the load capacity criteria for connectors in end spacer blocks, and the slenderness ratio limits for individual members (ℓ_1/d_1) and for the middle spacer block (ℓ_3/d_1) have been continued through to the 1991 edition.

15.2.1.1 In the design of spaced columns, the allowable capacity for an individual member is determined in accordance with the provisions of 15.2 and other applicable provisions of the Specification and then the associated design load for each member is summed to obtain the allowable load for the column. It is to be understood that the actual compression stress parallel to grain, f_c, on the members of the spaced column is not to exceed the allowable compression design value parallel to grain, F_c', for these members based on all provisions of 3.6 and 3.7 except as modified or extended by the provisions of 15.2. The net section requirements of 3.6.3 are to be applied to the members of spaced columns.

15.2.1.2 The advantage of a spaced column is the increase in the critical buckling design value for compression members obtained by the partial end fixity of the individual members. This increase in capacity, 2-1/2 or 3 times the value for a simple solid column with the same slenderness ratio, applies only to buckling in the direction perpendicular to the wide face of the members (buckling limited by the ℓ_1/d_1 ratio). If there was no slip in the end connections and full fixity of the ends were provided by the end block fastenings, the buckling stress would be 4 times that of a simple solid column because of the 50 percent reduction in effective column length (178).

The increase in the critical buckling stress associated with the ℓ_1/d_1 slenderness ratio obtained through the use of spaced column design may make capacity in the direction parallel to the wide face of the members (buckling associated with the ℓ_2/d_2 ratio) the limiting case. The allowable compression design value parallel to grain in this direction is not affected by spacing the individual members and, therefore, must be checked in accordance with 3.7.

15.2.2-Spacer and End Block Provisions

15.2.2.1 Where more than one spacer block is used, the distance ℓ_3 (see Figure 15A) is the distance from the center of one spacer block to the centroid of the connectors in the nearest end block.

15.2.2.2 Spacer blocks located within the middle one-tenth of the column length are not required to be joined to the compression members by split ring or shear plate connectors. Such blocks should be fastened through spiking, bolting or other means to assure the compression members maintain their spacing under load (62). A web member joined by connectors to two chords making up a spaced column may be considered a spacer block.

Where it is not feasible to use a single middle spacer block, two or more spacer blocks joined to compression members by split ring or shear plate connectors may be required to meet the ℓ_3/d_1 ratio limit of 40 (see 15.2.3.2). Connectors used in such spacer blocks must meet the same requirements as those applicable to end blocks and the distance between two adjacent spacer blocks is not to exceed one-half the distance between the centroids of connectors in the end blocks. Connectors are required for spacer blocks not located in the middle of the column length to provide the shear resistance necessary to assure the two members act as a unit under load. Use of connectors to join multiple spacer blocks to compression members has been a continuous requirement since the 1944 edition.

15.2.2.3 Spaced columns are used as compression chords in bowstring and other large span trusses (178). In this case, the web members of the truss serve as the end blocks. The distance between panel points which are laterally supported is taken as the length of such columns.

Until the 1962 edition, spaced-column web members were specifically provided for in the Specification by considering joints at the tension chord to be stayed laterally by the tautness of the tension chord and by the lateral bracing customarily used between trusses. Under present provisions, spaced-column web members may be designed using the procedures of 15.2 if the joints at both ends of the web member are laterally supported.

15.2.2.4 Prior to the 1962 edition, end and spacer blocks were permitted to have a thickness down to one-half that of the individual compression members if the length of the blocks was made inversely proportional to the thickness in relation to the required length of a full-thickness block. Since 1962, the thickness of end and spacer blocks is required to be equal to or larger than the thickness of the compression members (62).

It is to be noted that the length of end blocks and spacer blocks located at other than mid-length of the column should be sufficient to meet the end distance requirements for split ring or shear plate connectors given in Part X of the Specification. In this regard, the load on the connectors in the end blocks shall be considered applied in either direction parallel to the longitudinal axes of the compression members.

15.2.2.5 Connectors used in spaced columns are designed to restrain differential displacement between the individual compression members. Since the forces causing differential movement decrease as the ℓ/d of the individual members decrease, connector design value requirements vary with slenderness ratio (62).

The equations for end spacer block constants in 15.2.2.5 are based on K_S of zero when $\ell_1/d_1 \leq 11$ and a K_S equal to one-fourth a clear wood or basic compression design value parallel to grain for the species group when ℓ_1/d_1 is ≥ 60 (62). The equations give K_S values for intermediate slenderness ratios based on linear interpolation between these limits.

Except for modifications in load duration adjustment after World War II and conversion to a normal loading basis (see Commentary for 2.3.2), the limiting K_S values of 468, 399, 330 and 261 for species groups

A, B, C and D (defined in Table 10A of the specification), respectively, have remained unchanged since the 1944 edition. These values represent one-fourth the normal load, unseasoned basic compression design value parallel to grain applicable to representative species in each group in 1955 (62). Index basic compression design values parallel to grain used were dense Douglas fir and dense southern pine for Group A, Douglas fir and southern pine for Group B, western hemlock for Group C, and white firs-balsam fir for Group D.

The connector or connectors on each face of each end spacer block should be able to carry a load equal to the cross-sectional area of one of the individual compression members (without reduction for cuts made to receive connectors) times the end spacer block constant, K_S.

15.2.3-Column Stability Factor, C_P

15.2.3.1 Effective column length for spaced columns is determined in accordance with Figure 15A and adjusted by any applicable buckling length coefficient, K_e, greater than one as specified in Appendix G. It is to be noted that ℓ_1 is the distance between points of lateral support restraining movement perpendicular to the wide faces of the individual members, and ℓ_2 is the distance between points of lateral support restraining movement parallel to the wide faces of the individual members. ℓ_1 and ℓ_2 are not necessarily equal.

15.2.3.2 The slenderness ratio limits for spaced columns have been part of the Specification since the 1944 edition. The limit of 80 on the slenderness ratio ℓ_1/d_1 for the individual members is a conservative good practice recommendation that effectively provides a safety factor on the spaced column fixity coefficient, K_x. The limit of 50 on the slenderness ratio ℓ_2/d_2 is the limit applied to simple solid columns (see 3.7.1.4). The limit of 40 on the ℓ_3/d_1 ratio also is a conservative good practice recommendation to assure the length between end and spacer blocks in a spaced column is not a controlling factor in the column design.

15.2.3.3 The column stability factor for an individual member in a spaced column is calculated using the slenderness ratio ℓ_1/d_1 and the same equation as that applicable to simple solid columns (see 3.7.1.5) except that the critical buckling design value for compression, F_{cE}, is modified by the spaced column fixity coefficient, K_x.

The actual compression stress parallel to grain, f_c, calculated by dividing the total load on the spaced column by the sum of the cross-sectional areas of the individual members is checked against the product (F_c') of the column stability factor (C_P), the tabulated compression design value parallel to grain (F_c) and all other applicable adjustment factors (see 2.3). If connectors are required to join spacer (interior) blocks to individual members, and such blocks are in a part of the column that is most subject to potential buckling, f_c is to be calculated using the reduced or net section area remaining at the connector location (see 3.1.2) when comparing with the C_P adjusted allowable compression design value parallel to grain, F_c'

In all spaced-column designs, the actual compression stress parallel to grain, f_c, based on the net section area of the individual members at the end blocks is checked against the product of the tabulated compression design value parallel to grain and all applicable adjustment factors except the column stability factor (see 3.6.3).

15.2.3.4 Use of the lesser allowable compression design value parallel to grain, F_c', for a spaced column having members of different species or grades to all members is a conservative good practice recommendation introduced in the 1977 edition. Where the design involves the use of compression members of different thicknesses, the F_c' value for the thinnest member is to be applied to all other members.

15.2.3.5 The actual compression stress parallel to grain, f_c, in spaced columns also is to be checked in all cases against the allowable compression design value parallel to grain, F_c', based on the slenderness ratio ℓ_2/d_2 and a C_P factor calculated in accordance with the provisions of 3.7 without use of the spaced column fixity coefficient, K_x. Use of connectors to join individual compression members through end blocks increases the load carrying capacity of spaced columns only in a direction perpendicular to the wide face of the members. When the ratio of the width to thickness of the individual compression members is less than the square root of the spaced column fixity coefficient, K_x, the allowable compression stress parallel to grain, F_c', based on the slenderness ratio ℓ_2/d_2 will control.

15.2.3.6 (See Commentary for 3.7.1.6)

15.2.3.7 Design provisions for spaced beams joined by end blocks and connectors are not included in the Specification. The beam-column equations of 3.9 therefore apply only to those spaced columns that are subject to loads on the narrow edges of the members that cause bending in a plane parallel to their wide face.

15.3-BUILT-UP COLUMNS

Background

As with spaced columns, built-up columns obtain their efficiency by increasing the buckling resistance of the individual laminations. The closer the laminations of a mechanically fastened built-up column deform together (the smaller the amount of slip occurring between laminations) under compressive load, the greater the relative capacity of that column compared to a simple solid column of the same slenderness ratio made with the same quality of material.

Prior to the 1991 edition, no specific provisions for the design of nailed or bolted layered (all laminations of same face width) columns were included in the Specification. However, from 1944 through the 1986 editions, the Specification referenced guidelines (57,62) for the design of built-up columns made with pieces connected by nails, bolts or other mechanical fastenings (57,62). The column configurations covered by these guidelines were rectangular sections consisting of either laminations boxed around a solid core or parallel laminations with edge cover plates.

Based on tests of columns of various lengths (156,157), the referenced guidelines expressed the capacity of the two equivalent column types as a percentage of the strength of one-piece columns made with material of the same grade and species. Such efficiencies ranged from a value of 82 percent at an ℓ/d ratio of 6, decreasing to a low of 65 percent at an ℓ/d of 18 and then increasing to 82 percent at an ℓ/d of 26. Although the test columns were made with certain specific fastening schedules (156), the guidelines provided no information on the spacing and number of fastenings required to achieve the indicated efficiencies.

In 1977, new methodology for the design of layered, spaced and braced columns made with various types of mechanical fasteners was developed in Canada (111). The methodology enables the determination of the strength of any built-up column on the basis of the slip between members of the column in both the elastic and inelastic ranges. The theoretical formulas were verified through extensive testing including 400 column tests and evaluation of the load-slip properties of 250 different types of connections. The formulas describe column buckling behavior using the tangent modulus theory expression proposed by a Finnish researcher in 1956 and which is now incorporated in the Specification as the general design procedure for all types of wood columns (see Commentary for 3.7.1.5). The formulas are entered with fastener load-slip values based on beam-on-elastic-foundation principles (99).

The comprehensive design methodology for built-up layered columns was simplified in 1989 to a form that permitted codification of the procedures for these members in design specifications (110). The provisions for built-up columns in 15.3 of the Specification are based on this simplified methodology.

15.3.1-General

The provisions of 15.3 apply only to layered columns in which the laminations are of the same width and are unjointed. The limitations on number of laminations are based on the range of columns that were tested in the original research (111) that met the connection requirements of 15.3.3 and 15.3.4. The minimum lamination thickness requirement assures use of lumber for which approved design values are available in the Specification.

15.3.2-Column Stability Factor, C_p

Provisions in 15.3.2 are the same as those applicable to simple solid columns in 3.7.1 except for the addition of the column stability coefficients, K_f, in equation 15.3-1.

When nailed in accordance with the provisions of 15.3.3, the capacity of built-up columns has been shown to be more than 60 percent of that of an equivalent simple solid column at all ℓ/d ratios (110). Efficiencies are higher for conforming columns in the shorter ($\ell/d < 15$) and longer ($\ell/d > 30$) slenderness ratio ranges than those for columns in the intermediate range.

The efficiency of bolted built-up columns conforming to the connection requirements of 15.3.4 is more than 75 percent for all ℓ/d ratios (110). As with nailed columns, efficiencies of short and long bolted built-up columns are higher than those for intermediate ones. The greater efficiency of bolted compared to nailed columns is reflective of the higher load-slip moduli obtainable with the former.

In accordance with 3.7.1.3, Equation 15.3-1 is entered with a value of F_{cE} based on the larger of ℓ_{e1}/d_1 or ℓ_{e2}/d_2, where d_2 is the dimension of the built-up member across the weak axis of the individual laminations (sum of the thicknesses of individual laminations). Research (110) has shown that buckling about the weak axis of the individual laminations is a function of the amount of slip and load transfer that occur at fasteners between laminations. When the controlling slenderness ratio is the strong axis of the individual laminations, ℓ_{e1}/d_1, then $K_f = 1.0$. It is also necessary to compare C_p based on ℓ_{e1}/d_1 and $K_f = 1.0$ with C_p based on ℓ_{e2}/d_2 and $K_f = 0.6$ or 0.75

to determine the allowable compression design value parallel to grain, F_c'.

15.3.3-Nailed Built-up Columns

15.3.3.1 Nailing requirements (a), (b) and (g) and the maximum spacing requirements of (d) and (e) are based on the conditions for which the column stability coefficient, K_f, of 60 percent was established (110). The maximum spacing between nails in a row of 6 times the thickness of the thinnest lamination minimizes the potential for buckling of the individual laminations between connection points. End, edge and minimum spacing requirements are good practice recommendations for preventing splitting of members (41) and for assuring fasteners are well distributed across and along the face of the laminations.

The requirement for adjacent nails to be driven from opposite sides of the column applies to adjacent nails aligned both along the grain of the laminations and across their width.

In the nailing requirements of 15.3.3.1, a nail row refers to those nails aligned parallel to the grain of the laminations and in the direction of the column length. Where only one longitudinal row of nails is required, such nails are required to be staggered along either side of the center line of the row. Adjacent off set nails in such a configuration should be driven from opposite faces.

Where three rows of nails are required by spacing and edge distance requirements, nails in adjacent rows are to be staggered and adjacent nails beginning with the first in each row driven from opposite sides as if nails were aligned across the face of the laminations.

15.3.4-Bolted Built-up Columns

15.3.4.1 Maximum spacing limits for bolts and rows, and number of row requirements in (d), (e) and (g), respectively, are based on conditions for which the bolted built-up column efficiency factor, K_f, was established (110). Maximum end distance limits in (c) are good practice recommendations (41) to assure end bolts are placed close to the ends of the column where interlaminar shear forces are largest. Minimum end distance, spacing between adjacent bolts in a row, spacing between rows and edge distance in (c), (d), (e) and (f) correspond to provisions governing bolted joints in 8.5.

As with nailed columns, a bolt row refers to those nails aligned parallel to the grain of the laminations and in the direction of the column length. The maximum spacing of bolts in a row of six times the lamination thickness minimizes the potential for buckling of individual laminations between connection points.

15.4-WOOD COLUMNS WITH SIDE LOADS AND ECCENTRICITY

15.4.1-General Equations

Equations for wood columns with side loads and eccentricity have been included in the Specification since the 1944 edition. Based on theoretical analyses (223), these equations remained essentially unchanged through the 1986 edition except for changes made in column design provisions and the interaction equations over this period (see Commentary for 3.7.1.5 and 3.9.2).

The equation in 15.4.1 for combined bending and eccentric axial compression loads reflects the introduction of the new continuous column equation and the new beam-column equation in the 1991 edition. The equation is an expansion of the interaction equation given in 3.9.2 to the general case of any combination of side loads, end loads and eccentric end loads (229).

For the case of a bending load on the narrow face and an eccentric axial load producing a moment in the same direction as the bending load, the general interaction equation in 15.4-1 reduces to

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1}+f_c(6e_1/d_1)[1+0.234(f_c/F_{cE1})]}{F_{b1}'[1-(f_c/F_{cE1})]} \leq 1.0 \qquad \text{(C15.4-1)}$$

or

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1}+f_c(6e_1/d_1)[1.234-0.234\,C_{m1}]}{C_{m1}F_{b1}'} \leq 1.0 \qquad \text{(C15.4-2)}$$

where:

e_1 = eccentricity

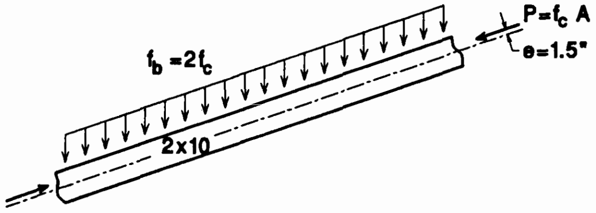
C_{m1} = moment magnification factor = $1-f_c/F_{cE1}$

and other symbols as defined in the Commentary for 3.9.2

For a long column ($J = 1$), the comparable equation in the 1986 and earlier editions was

Example C15.4-1

A No. 1 Douglas Fir-Larch 2×10 is used as the upper chord of a roof truss. The truss is designed such that the flexural stress in the member from roof loads (DL+SL) is twice the axial compressive stress in the member from the truss reactions from roof and ceiling loads (DL+SL). At the panel points, the axial force acts eccentrically at a point 1.5 in. above the center of the width of the member. The top edge of the chord is laterally supported with ℓ_{e1} = 94 in. and ℓ_{e2} = 0 in. Determine the allowable axial force in the member based on the interaction of bending and compression.



F_b = 1000 psi C_F = 1.1 C_D = 1.15 (Table 4A)
F_c = 1450 psi C_F = 1.0 E = 1,700,000 psi
A = 13.88 in^2 S = 21.39 in^3

Allowable Compression Design Value Parallel to Grain (3.6, 3.7)

$F_c^* = F_cC_DC_F$ = (1450)(1.15)(1.0) = 1668 psi (3.7.1.5)

$\ell_{e2}/d_2 = 0$ (fully supported)

$\ell_e/d = \ell_{e1}/d_1 = (94)/(9.25) = 10.16 < 50$

$K_{cE} = 0.3$

$$F_{cE} = \frac{K_{cE}E'}{(\ell_e/d)^2} = \frac{(0.3)(1{,}700{,}000)}{(10.16)^2} = 4940 \text{ psi}$$

$$C_P = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$= \frac{1+4940/1668}{2(0.8)} - \sqrt{\left[\frac{1+4940/1668}{2(0.8)}\right]^2 - \frac{4940/1668}{0.8}}$$

$= 0.918$

$F_c' = F_cC_DC_FC_P$ = (1450)(1.15)(1.0)(0.918) = 1530 psi

Allowable Bending Design Value (3.3.3)

Full lateral support along narrow face, $C_L = 1.0$ (3.3.3.3)

$F_b' = F_bC_DC_LC_F$ = (1000)(1.0)(1.15)(1.1) = 1265 psi

Combined Bending and Axial Compression (15.4.1)

No minor axis bending

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1} + f_c(6e_1/d_1)[1+0.234(f_c/F_{cE1})]}{F_{b1}'\,[1-(f_c/F_{cE1})]} \le 1.0$$

For $f_{b1} = 2f_c$, $e = 1.5$ in., $d_1 = 9.25$ in.

$$\left(\frac{f_c}{1530}\right)^2 + \frac{2f_c + (f_c)(6(1.5)/9.25)[1+0.234(f_c/4940)]}{(1265)[1-(f_c/4940)]}$$

Solving for f_c by iteration

f_c = 368.75 psi

$f_b = 2f_c$ = 2(368.75) = 737.5 psi

$P_{allowable} = f_cA$ = (368.75)(13.88) = 5118 lb

Maximum axial compression force in 2x10 chord member based on interaction of bending and compression = 5118 lb

$$\frac{f_c}{F_c'} + \frac{f_{b1} + f_c(7.5e_1/d_1)}{F_{b1}' - f_c} \le 1.0 \qquad \text{(C15.4-3)}$$

The new and old equations for this loading case differ by the coefficient applied to the moment due to the eccentric load, or

$$6\,(1.234 - 0.234\,C_{m1}) \quad vs. \quad 7.5 \qquad \text{(C15.4-4)}$$

as well as the previously discussed (Commentary 3.9.2) difference in the exponent applied to the compression ratio term (2nd compared to 1st power) and the moment magnification factor ($1-f_c/F_{cE1}$ compared to $1-Jf_c/F_{b1}'$). Example C15.4-1 illustrates the use of the new provisions.

For the case of a bending load on the narrow face and an eccentric (e_2) axial load producing a moment perpendicular to the plane of bending due to the edgewise load, the applicable interaction equation is

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1}}{C_{m1}F_{b1}'} + \frac{f_c(6e_2/d_2)[1.234 - 0.234\,C_{m2}]}{C_{m2}F_{b2}'} \le 1.0 \qquad \text{(C15.4-5)}$$

where:

C_{m1} = moment magnification factor = $1-f_c/F_{cE1}$

C_{m2} = moment magnification factor
= $1 - f_c/F_{cE2} - (f_{b1}/F_{bE})^2$

or in expanded form

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1}}{F_{b1}'[1 - (f_c/F_{cE1})]} + \qquad \text{(C15.4-6)}$$

$$\frac{f_c(6e_2/d_2)[1 + 0.234(f_c/F_{cE2}) + 0.234(f_{b1}/F_{bE})^2]}{F_{b2}'[1 - (f_c/F_{cE2}) - (f_{b1}/F_{bE})^2]} \leq 1.0$$

A comparable equation for this loading case was not provided in earlier editions.

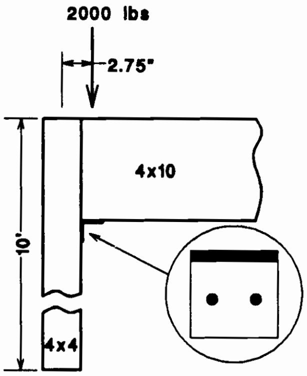
15.4.2-Columns with Side Brackets

The procedure for calculating the portion of an axial load applied through a bracket that is assumed to act as a side load at mid height of the column is based on early recommendations (57) and has been a provision of the Specification since the 1944 edition. The calculated side load, P_s, acting at midspan is considered to produce a moment at this location ($P_s \ell/4$) equal to the moment produced by the load on the bracket, Pa, times three-fourths the distance from the top of the bracket to the base of the column (ℓ_p) divided by the column length (ℓ).

When the bracket is at the top of the column, results obtained by entering Specification Equation 15.4-1 (or Equation 3.9-3) with a concentric axial load and the calculated side load, P_s, will give a 25 percent lower combined stress index than that obtained from the eccentric axial end load formula, Specification Equation 15.4-2. This difference is a result of the latter being based on the assumption of eccentric loads on both ends of the column (constant moment along the length of the column) whereas the procedure in section 15.4.2 assumes the moment due to the bracket load decreases linearly from the point of application to zero at the column base. The procedure of section 15.4.2 may be used for those columns where the point of application of the eccentric axial load is outside the column cross-section (see Example C15.4-2).

Example C15.4-2

A 4×10 roof rafter is supported on an L2x2x1/4 steel side bracket (3.5 in. long), connected to a 10 ft long 4×4 column using two 5/8 in. bolts. There is a 2000 lb reaction at the bracket from the rafter due to dead load plus snow loads on the roof. Both members are No. 2 Douglas Fir-Larch. Check the adequacy of the connection and joined members.



F_b = 875 psi C_F = 1.5 (4×4) C_D = 1.15 (Table 4A)

F_c = 1300 psi C_F = 1.15 (4×4) E = 1,600,000 psi

F_v = 95 psi $F_{c\perp}$ = 625 psi

Check Rafter at Member End

Assume member is adequate for bending and deflection

Shear (3.4)

$F_v' = F_v C_D C_M C_t$ = (95)(1.15)(1.0)(1.0) = 109 psi

$V_{reaction}$ = 2000 lb

$$f_v = \frac{3V}{2bd} = \frac{(3)(2000)}{(2)(3.5)(9.25)} = 93 \text{ psi} < F_v' = 109 \text{ psi} \quad \textbf{ok}$$

Bearing (3.10.2)

$F_{c\perp}' = F_{c\perp} C_M C_t C_b$ = (625)(1.0)(1.0)(1.0) = 625 psi

Reaction = 2000 lb

Bearing area of bracket = (2.0)(3.5) = 7.0 in^2

$f_{c\perp}$ = (2000)/(7.0) = 286 psi < $F_{c\perp}'$ = 625 psi **ok**

Rafter is adequate at member end

Check Bracket Bolts in Column

For 5/8-in. bolts (side by side) in single shear with t_m = 3.5 in. and t_s = 1/4 in.

$Z_{//}$ = 1130 lb/bolt (Table 8.2B)

$Z' = Z_{//} C_D C_g C_\Delta$ = (1130)(1.15)(1.0)(1.0) (7.3.1)

= 1300 lb

P = 2000 lb < nZ' = (2)(1300) = 2600 lb **ok**

Check Column by Two Methods (15.4.2)

1. **Assume eccentric load acts at the top end of the column** (15.4.2.1)

Compression (3.6, 3.7)

$F_c^* = F_c C_D C_F$ = (1300)(1.15)(1.15) = 1719 psi (3.7.1.5)

Check net section at bracket (no buckling) (3.6.3)

(cont.)

Example C15.4-2 (cont.)

$A_{gross} = (3.5)(3.5) = 12.25\ \text{in}^2$

$A_{net} = (3.5)(3.5 - 2(5/8 + 1/16)) = 7.44\ \text{in}^2$

$f_c = P/A_{net} = 2000/7.44$

$= 269\ \text{psi} < F_c^* = 1719\ \text{psi}$ **ok**

Check gross section (potential buckling) (3.6.3, 3.7.1)

$\ell_e/d = \ell_1/d_1 = \ell_2/d_2 = (10)(12)/(3.5) = 34.3 < 50$

$K_{cE} = 0.3$

$$F_{cE} = \frac{K_{cE}E'}{(\ell_e/d)^2} = \frac{(0.3)(1{,}600{,}000)}{(34.3)^2} = 408\ \text{psi}$$

$$C_P = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$= \frac{1+408/1719}{2(0.8)} - \sqrt{\left[\frac{1+408/1719}{2(0.8)}\right]^2 - \frac{408/1719}{0.8}}$$

$= 0.2244$

$F_c' = F_c C_D C_F C_P = (1300)(1.15)(1.15)(0.2244)$ (2.3.1)

$= 386\ \text{psi}$

$f_c = P/A_g = 2000/12.25 = 163\ \text{psi} < F_c' = 386\ \text{psi}$ **ok**

Bending (3.3)

Since $d = b = 3.5$ in., $C_L = 1.0$

$F_b' = F_b C_D C_L C_F = (875)(1.15)(1.0)(1.5) = 1509\ \text{psi}$ (2.3.1)

Check net section at bracket (3.2.1)

eccentricity = 3.5/2 + 2.0/2 = 2.75 in.

$S_{net} = (3.5 - 2(5/8 + 1/16))(3.5)^2/6 = 4.339\ \text{in}^3$

$M_{bracket} = Pe - Ped_{rafter}/\ell$

$= (2000)(2.75) - (2000)(2.75)(9.25)/(10)(12)$

$= 5076$ in-lb

$f_b = M_{bracket}/S_{net} = (5076)/(4.339)$

$= 1170\ \text{psi} < F_b' = 1509\ \text{psi}$ **ok**

Check gross section (at end)

$S_{gross} = 7.146\ \text{in}^3$

$M_{end} = Pe = (2000)(2.75) = 5500$ in-lb

$f_b = M_{end}/S_{gross} = (5500)/(7.146)$

$= 770\ \text{psi} < F_b' = 1509\ \text{psi}$ **ok**

Combined Bending and Axial Compression (15.4.1)

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_c(6e_1/d_1)[1+0.234(f_c/F_{cE1})]}{F_{b1}'\,[1-(f_c/F_{cE1})]} \le 1.0$$

$$\left(\frac{163}{386}\right)^2 + \frac{(163)(6(2.75)/3.5)[1+0.234(163/408)]}{(1509)[1-(163/408)]}$$

$= 1.11 > 1.0$ **ng**

Column does not meet NDS provisions by Method 1

2. **Assume that the load on the bracket is replaced by the same load, centrally applied at the top of the column, plus a side load, P_s, applied at midheight (See Commentary for 15.4.2).** (15.4.2.2)

Compression (3.6, 3.7)

Same as Method 1

Bending (3.3)

Calculate side load (Eq. 15.4-3)

$P = 2000$ lb, a = eccentricity = 2.75 in.

$\ell = (10)(12) = 120$ in.

$\ell_p = \ell - d_{rafter} = 120 - 9.25 = 110.75$ in.

$P_s = 3Pa\ell_p/\ell^2$

$= (3)(2000)(2.75)(110.75)/(120)^2$

$= 127$ lb at $\ell/2 = 60$ in.

Since $d = b = 3.5$ in., $C_L = 1.0$

$F_b' = F_b C_D C_L C_F = (875)(1.15)(1.0)(1.5) = 1509\ \text{psi}$ (2.3.1)

Check gross section (at $\ell/2$)

$S_{gross} = 7.146\ \text{in}^3$

$M_{max} = P_s\ell/4 = (127)(120)/4 = 3810$ in-lb

$f_b = M_{max}/S_{gross} = (3810)/(7.146)$

$= 533\ \text{psi} < F_b' = 1509\ \text{psi}$ **ok**

Combined Bending and Axial Compression (3.9.2)

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1}}{F_{b1}'\,(1 - f_c/F_{cE1})} \le 1.0$$

$$\left(\frac{163}{386}\right)^2 + \frac{533}{1509\,(1 - 163/408)} = 0.77 < 1.0$$ **ok**

Column satisfies NDS provisions by Method 2